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DETERMINATION OF PROXIMITY OF ELLIPTICAL ORBITS OF CELESTIAL BODIES BY WAY OF ECCENTRIC ANOMALIES

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Summary. Presentation is given of general method of the calculus of proximity of elliptical orbits, with the eccentric anoma ies as basic variables. Equations are derived for the determination of points at the minimum mutual distance, irrespective of how great is the angle between the orbital planes. These equations being transcendent, their solution is performed by the method of successive approximations. One proceeds thereby from the approximate values of eccentric anomalies, which are fairly close to those searched for and their successive corrections are being determined. The procedure is very effective and suited to the serial investigation. It is illustrated by some examples of minor planet pairs.

J. Lazović, ODREĐIVANJE PROKSIMITETA ELIPTIČNIH PUTANJA NEBESKIH TELA POMOĆU EKSCENTRIČNIH ANOMALIJA — Prikazana je opšta metoda za račun proksimiteta eliptičnih putanja nebeskih tela sa ekscentričnim anomalijama kao osnovnim promenljivim. Izvedene su jednačine za određivanje tačaka s najmanjim međusobnim rastojanjem, nezavisno od veličine ugla između putanjskih ravni. Zbog transcendentnosti ovih jednačina, one se rešavaju metodom uzastopnih aproksimacija, polazeći od približnih vrednosti ekscentričnih anomalija, koje su dovoljno bliske traženim, i određuju se njihove uzastopne popravke. Postupak je vrlo efikasan i pogodan za serijsko ispitivanje, a ilustrovan je na primerima parova malih planeta.

In a previous paper of ours we derived formulae for the position determination of points of the least (proximity) distance of two celestial bodies, moving in elliptical orbits, using their true anomalies (Lazović, 1967). It was then an easy matter to calculate the proximity distance itself. In applying these formulae to the minor planets we obtained very interesting, in our view, results (Lazović, Kuzmanoski, 1978, 1979, 1980a, 1980b, 1981). Sitarski (1968), too, derived the formulae for the calculation of the minimum distance between two heliocentric orbits.

The solution of the proximity problem by way of the eccentric anomalies has been treated by Lazović (1964). However, as these results were not easily accessible to the broader public we are going to present them herein. Meanwhile significant contributions, dealing with the proximity determination have been realited, which will be referred to later. Through them the solution of our previous equations for the proximity positions has been facilitated. These equations will be now derived differently than in Lazović (1964). Simovljevitch (1974), too, proposed a method of the proximity determination by means of the eccentric anomalies.

Suppose two celestial bodies j and k, considered as material points, moving in elliptical heliocentric orbits with known elements. Then the heliocentric position vectors of these bodies as a function of their eccentric anomalies E_j and E_k

$$\mathbf{r}_i = (\cos E_i - e_i)\,\mathbf{A}_i + \sin E_i\,\mathbf{B}_i, \qquad {\scriptstyle i=j,\,k} \tag{1}$$

$$\left.\begin{array}{l} \mathbf{A}_i = a_i\,\mathbf{P}_i, \quad \mathbf{B}_i = b_i\,\mathbf{Q}_i, \quad |\mathbf{P}_i| = 1, \quad |\mathbf{Q}_i| = 1, \quad (\mathbf{P}_i\,\mathbf{Q}_i) = 0, \\ |\mathbf{A}_i| = a_i, \quad |\mathbf{B}_i| = b_i = a_i\sqrt{1-e_i^2} = a_i\cos\varphi_i, \quad e_i = \sin\varphi_i, \\ (\mathbf{A}_i\,\mathbf{A}_i) = A_i^2 = a_i^2, \quad (\mathbf{B}_i\,\mathbf{B}_i) = B_i^2 = b_i^2 = a_i^2(1-e_i^2), \quad (\mathbf{A}_i\,\mathbf{B}_i) = 0, \qquad {\scriptstyle i=j,\,k} \end{array}\right\} \tag{2}$$

The rectangular heliocentric ecliptic coordinates of the vector constants $\mathbf{P}_i$ and $\mathbf{Q}_i$ are expressed by the known formulae

$$\left.\begin{array}{l} P_{xi} = \cos\omega_i\cos\Omega_i - \sin\omega_i\sin\Omega_i\cos i_i, \\ P_{yi} = \cos\omega_i\sin\Omega_i + \sin\omega_i\cos\Omega_i\cos i_i, \\ P_{zi} = \sin\omega_i\sin i_i, \\ Q_{xi} = -\sin\omega_i\cos\Omega_i - \cos\omega_i\sin\Omega_i\cos i_i, \\ Q_{yi} = -\sin\omega_i\sin\Omega_i + \cos\omega_i\cos\Omega_i\cos i_i, \\ Q_{zi} = \cos\omega_i\sin i_i, \\ \qquad {\scriptstyle i=j,\,k.} \end{array}\right\} \tag{3}$$

From (2) and (3) ecliptic coordinates of the constant vectors $\mathbf{A}_i = \{A_{xi}, A_{yi}, A_{zi}\}$ and $\mathbf{B}_i = \{B_{xi}, B_{yi}, B_{zi}\}$ are also easily obtained. From (1) and (2) we derive the rectangular heliocentric ecliptic coordinates of our two celestial bodies as a function of their eccentric anomalies

$$\begin{array}{c} l_i = (\cos E_i - e_i)\,A_{li} + \sin E_i\,B_{li}, \\ l = x, y, z, \qquad {\scriptstyle i=j,\,k} \end{array} \tag{4}$$

If $\boldsymbol{\rho}$ is the relative position vector of body j relative to the body k

$$\boldsymbol{\rho} = \mathbf{r}_j - \mathbf{r}_k, \tag{5}$$

then the distance ρ between these two bodies can be calculated from its square

$$\begin{array}{l} \rho^2 = (\mathbf{r}_j - \mathbf{r}_k)^2 = (\mathbf{r}_j\,\mathbf{r}_j) - 2\,(\mathbf{r}_j\,\mathbf{r}_k) + (\mathbf{r}_k\,\mathbf{r}_k) = \\ \quad = (x_j - x_k)^2 + (y_j - y_k)^2 + (z_j - z_k)^2, \end{array} \tag{6}$$

accordingly as a function of two variables E_j and E_k.

The minimum distance between the two bodies $\min\rho = \rho_{\min}$ takes place for $\min\frac{1}{2}\rho^2$. Hence, by introducing the function

$$F(E_j, E_k) = \frac{1}{2}\rho^2,$$

the equations for the determination of points of the considered orbits at minimum mutual distance are obtained from the conditions

$$\frac{\partial F}{\partial E_j}=0, \qquad \frac{\partial F}{\partial E_k}=0 \tag{7}$$

and

$$U=\frac{\partial^2 F}{\partial E_j^2}\cdot\frac{\partial^2 F}{\partial E_k^2}-\left(\frac{\partial^2 F}{\partial E_j\,\partial E_k}\right)^2>0, \text{ and } \frac{\partial^2 F}{\partial E_j^2}\left(\text{or } \frac{\partial^2 F}{\partial E_k^2}\right)>0. \tag{8}$$

By denoting

$$\mathbf{R}_i=\frac{\partial \mathbf{r}_i}{\partial E_i}, \qquad {}_{i=j,\,k}$$

we obtain from (1)

$$\mathbf{R}_i=-\sin E_i\,\mathbf{A}_i+\cos E_i\,\mathbf{B}_i. \tag{9}$$

From (6) and (5) we derive

$$f=f(E_j, E_k)=\frac{\partial F}{\partial E_j}=(\rho\,\mathbf{R}_j), \quad g=g(E_j, E_k)=\frac{\partial F}{\partial E_k}=-(\rho\,\mathbf{R}_k), \tag{10}$$

whereby the designations of the functions f and g are also introduced.

On taking

$$\frac{\partial\rho}{\partial E_j}=\mathbf{R}_j, \qquad \frac{\partial\rho}{\partial E_k}=-\mathbf{R}_k,$$

we obtain

$$\left.\begin{aligned} f'_{E_j}&=\frac{\partial f}{\partial E_j}=\frac{\partial^2 F}{\partial E_j^2}=(\mathbf{R}_j\,\mathbf{R}_j)+\left(\rho\,\frac{\partial \mathbf{R}_j}{\partial E_j}\right),\\ g'_{E_k}&=\frac{\partial g}{\partial E_k}=\frac{\partial^2 F}{\partial E_k^2}=(\mathbf{R}_k\,\mathbf{R}_k)-\left(\rho\,\frac{\partial \mathbf{R}_k}{\partial E_k}\right),\\ f'_{E_k}&=\frac{\partial f}{\partial E_k}=\frac{\partial^2 F}{\partial E_j\,\partial E_k}=\frac{\partial g}{\partial E_j}=g'_{E_j}=-(\mathbf{R}_j\,\mathbf{R}_k). \end{aligned}\right\} \tag{11}$$

From (9) we have

$$\frac{\partial \mathbf{R}_i}{\partial E_i}=\frac{\partial^2 \mathbf{r}_i}{\partial E_i^2}=-\cos E_i\,\mathbf{A}_i-\sin E_i\,\mathbf{B}_i, \qquad {}_{i=j,\,k.} \tag{12}$$

The equations of conditions (7) for the proximity, having regard to (10), (5) and (9) become

$$f(E_j, E_k)=(\mathbf{r}_j-\mathbf{r}_k)\cdot(-\sin E_j\,\mathbf{A}_j+\cos E_j\,\mathbf{B}_j)=0,$$

$$g(E_j, E_k)=(\mathbf{r}_j-\mathbf{r}_k)\cdot(\sin E_k\,\mathbf{A}_k-\cos E_k\,\mathbf{B}_k)=0.$$

Upon substituting in these equations the expressions (1) and (2) and performing scalar multiplications of the corresponding vectors we obtain

$$f(E_j, E_k) = (H_j + D\sin E_k + C\cos E_k)\sin E_j + (K_j - G\sin E_k - \Phi\cos E_k)\cos E_j - \\ - S_j \sin E_j \cos E_j = 0, \qquad (13)$$

$$g(E_j, E_k) = (H_k + \Phi\sin E_j + C\cos E_j)\sin E_k + (K_k - G\sin E_j - D\cos E_j)\cos E_k - \\ - S_k \sin E_k \cos E_k = 0,$$

where the constant quantities, in view of (2), are denoted

$$\left.\begin{aligned} C &= (\mathbf{A}_j\,\mathbf{A}_k) = a_j\, a_k\,(P_{xj}\,P_{xk} + P_{yj}\,P_{yk} + P_{zj}\,P_{zk}),\\ D &= (\mathbf{A}_j\,\mathbf{B}_k) = a_j\, b_k\,(P_{xj}\,Q_{xk} + P_{yj}\,Q_{yk} + P_{zj}\,Q_{zk}),\\ \Phi &= (\mathbf{A}_k\,\mathbf{B}_j) = a_k\, b_j\,(P_{xk}\,Q_{xj} + P_{yk}\,Q_{yj} + P_{zk}\,Q_{zj}),\\ G &= (\mathbf{B}_j\,\mathbf{B}_k) = b_j\, b_k\,(Q_{xj}\,Q_{xk} + Q_{yj}\,Q_{yk} + Q_{zj}\,Q_{zk}),\\ &H_j = a_j^2\, e_j - Ce_k, \quad H_k = a_k^2\, e_k - Ce_j, \quad K_j = \Phi e_k,\\ &K_k = De_j, \quad S_j = a_j^2\, e_j^2, \quad S_k = a_k^2\, e_k^2\,. \end{aligned}\right\} \qquad (14)$$

The coefficients (14) are calculated from the known orbital elements of the celestial bodies under consideration by means of the expressions (3).

The equations (13) can be written in an abreviated form

$$\left.\begin{aligned} f(E_j, E_k) &= X_k \sin E_j + Y_k \cos E_j - S_j \sin E_j \cos E_j = 0,\\ g(E_j, E_k) &= X_j \sin E_k + Y_j \cos E_k - S_k \sin E_k \cos E_k = 0, \end{aligned}\right\} \qquad (15)$$

where

$$\left.\begin{aligned} X_j &= X_j(E_j) = H_k + \Phi\sin E_j + C\cos E_j,\\ X_k &= X_k(E_k) = H_j + D\sin E_k + C\cos E_k,\\ Y_j &= Y_j(E_j) = K_k - G\sin E_j - D\cos E_j,\\ Y_k &= Y_k(E_k) = K_j - G\sin E_k - \Phi\cos E_k. \end{aligned}\right\} \qquad (16)$$

Therefore, for the determination of the eccentric anomalies E_j and E_k, corresponding to the positions of the shortest mutual distance of the bodies considered, we obtained two trigonometric equations (15), in their form they are equal to those in Lazović (1964). They apply to the general case in the calculus of elliptical orbit proximities, independently of how great is the angle between the orbital planes.

Let us now determine the expressions, appearing in (8), which we will need later. Note that because of the notations in (11) we can have in a new form

$$U = f'_{E_j} g'_{E_k} - f'_{E_k} g'_{E_j} = f'_{E_j} g'_{E_k} - (f'_{E_k})^2 = f'_{E_j} g'_{E_k} - (g'_{E_j})^2. \qquad (17)$$

The partial derivatives (11), because of (1), (5), (9) and (12), assume the forms

$$f'_{E_j} = X_k \cos E_j - Y_k \sin E_j + S_j(\sin^2 E_j - \cos^2 E_j),$$

$$g'_{E_k} = X_j \cos E_k - Y_j \sin E_k + S_k(\sin^2 E_k - \cos^2 E_k), \qquad (18)$$

$$f'_{E_k} = g'_{E_j} = -C\sin E_j \sin E_k + D\sin E_j \cos E_k + \Phi\cos E_j \sin E_k - G\cos E_j \cos E_k.$$

From above presentation we see that our basis for the solution of the proximity of two bodies j and k is constituted by the functions of eccentric anomalies

$$f(E_j, E_k) = \frac{1}{2}\frac{\partial \rho^2}{\partial E_j}, \qquad g(E_j, E_k) = \frac{1}{2}\frac{\partial \rho^2}{\partial E_k},$$

which are vanishing for the proximity position values of E_j and E_k. The equations $f(E_j, E_k) = 0$, $g(E_j, E_k) = 0$ in the form (15) are transcendent. In order to solve them, we expand in series the functions f and g, proceeding from the known approximate values of the eccentric anomalies of the proximity positions E_{j0} and E_{k0},

$$f = f_0 + f'_{E_{j0}} \Delta E_j + f'_{E_{k0}} \Delta E_k + \ldots,$$

$$g = g_0 + g'_{E_{j0}} \Delta E_j + g'_{E_{k0}} \Delta E_k + \cdots,$$

where

$$f_0 = f(E_{j0}, E_{k0}), \quad g_0 = g(E_{j0}, E_{k0}), \quad f'_{E_{j0}} = \left(\frac{\partial f}{\partial E_j}\right)_{E_j = E_{j0},\, E_k = E_{k0}},$$

$$f'_{E_{k0}} = \left(\frac{\partial f}{\partial E_k}\right)_{E_j = E_{j0},\, E_k = E_{k0}}, \qquad g'_{E_{j0}} = \left(\frac{\partial g}{\partial E_j}\right)_{E_j = E_{j0},\, E_k = E_{k0}},$$

$$g'_{E_{k0}} = \left(\frac{\partial g}{\partial E_k}\right)_{E_j = E_{j0},\, E_k = E_{k0}}.$$

By equating to zero the first three terms in both series we get the equations

$$\left.\begin{aligned} f_0 + f'_{E_{j0}} \Delta E_{j0} + f'_{E_{k0}} \Delta E_{k0} = 0, \\ g_0 + g'_{E_{j0}} \Delta E_{j0} + g'_{E_{k0}} \Delta E_{k0} = 0, \end{aligned}\right\} \tag{19}$$

whose solutions make it possible to find the first corrections ΔE_{j0} and ΔE_{k0} to the starting values of the eccentric anomalies. By so doing we obtain new values $E_{j1} = E_{j0} + \Delta E_{j0}$ and $E_{k1} = E_{k0} + \Delta E_{k0}$ enabling us to derive in an analogous way new corrections and to get even more approximate values of the searched for anomalies and so forth. Accordingly, by the method of successive approximations, after performing calculations of n pairs of corrections — an operation reiterated until such two successive approximations are performed that yield the same value of the eccentric anomaly with the necessary accuracy corresponding to one and the same body — the solutions of the system (15) are obtained in the form

$$E_{jn} = E_{j(n-1)} + \Delta E_{j(n-1)}, \qquad E_{kn} = E_{k(n-1)} + \Delta E_{k(n-1)}, \tag{20}$$
$$n = 1, 2, 3, \ldots .$$

This procedure is rapidly convergent as the required solutions are, in the practice, usually obtained after only two approximations. The corrections to the eccentric anomalies, expressed in radians, are furnished by the solution of the system of equations, which is similar to the system (19), on putting the index $(n-1)$ in place of 0. They are

$$\left.\begin{aligned} \Delta E_{j(n-1)} &= \frac{g_{n-1} f'_{E_{k(n-1)}} - f_{n-1} g'_{E_{k(n-1)}}}{U_{n-1}}, \\ & \qquad\qquad\qquad\qquad n=1,2,3,\ldots, \\ \Delta E_{k(n-1)} &= \frac{f_{n-1} g'_{E_{j(n-1)}} - g_{n-1} f'_{E_{j(n-1)}}}{U_{n-1}}. \end{aligned}\right\} \tag{21}$$

The denominator U_{n-1} is calculated by (17), taking $E_j = E_{j(n-1)}$ and $E_k = E_{k(n-1)}$. These values of the eccentric anomalies are used at calculating the corresponding values of the functions f and g and their partial derivatives f'_{E_i} and g'_{E_i}, $i = j, k$, by means of (2), (3), (14), (16), (15) and (18).

The necessary approximate values of the eccentric anomalies of the proximity positions E_{j0} and E_{k0} can be obtained either indirectly or directly. They can be calculated indirectly using the corresponding, previously determined, approximate values v_{j0} and v_{k0} of the true anomalies of the proximity positions, by means of the known relation

$$\operatorname{tg}\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\operatorname{tg}\frac{v}{2}. \tag{22}$$

In this, use is made of some of the procedures described in our earlier papers (Lazović, 1967, 1974, 1976, 1978, 1980), having regard to whether a pair of the quasicomplanar or non-quasicomplanar bodies is being treated. Preference is accorded to the formulae from the three last quoted papers. The quantities E_{j0} and E_{k0} can be determined directly using the formulae developed by Simovljevitch (1977). They are of the form

$$\left.\begin{aligned} & x_j = \frac{(\mathbf{P}_j\,\mathbf{R}_k)}{(\mathbf{Q}_j\,\mathbf{R}_k)}\sec\varphi_j, \qquad x_k = \frac{(\mathbf{P}_k\,\mathbf{R}_j)}{(\mathbf{Q}_k\,\mathbf{R}_j)}\sec\varphi_k, \\ & \operatorname{tg} L_i = x_i, \quad \sin(E_i + L_i) = e_i \sin L_i, \qquad i=j,k. \end{aligned}\right\} \tag{23}$$

It is to be noted that here the indeks $_0$ is omitted with E_i for the approximate values of the eccentric anomalies. The value of L_i is somewhere between 0° and 180? The scalar products of the corresponding pairs of unit vectors $\mathbf{P}_i$, $\mathbf{Q}_i$ and $\mathbf{R}_i$ can be deduced from their rectangular ecliptic coordinates (3) and

$$\left.\begin{aligned} R_{xi} &= \sin \Omega_i \sin i_i, \\ R_{yi} &= -\cos \Omega_i \sin i_i, \\ R_{zi} &= \cos i_i, \qquad i=j,k. \end{aligned}\right\} \tag{24}$$

$\mathbf{R}_i$ are unit vectors, perpendicular to the orbital planes of the bodies concerned, $\mathbf{R}_i = [\mathbf{P}_i\,\mathbf{Q}_i]$. In the paper of Simovljevitch just referred to, explanation is given of the way of obtaining, by these formulae, that solution which corresponds to the required proximity of the orbits.

The procedure of proximity determination of the elliptic orbits of the celestial bodies by way of eccentric anomalies, set out here, will be illustrated by two examples of minor planets pairs. One of pairs is characterised by minor planets moving in nearly the same plane, more exactly, the angle between their orbital planes is small. These pairs, or planetoid groups, were termed quasicomplanar planetoids by our professor V. V. Michkovitch (1974). The other pair is characterised by a large angle between their orbital planes, i. e. it is a non-quasicomplanar one.

To the first example belongs the quasicomplanar pair of the minor planets (589) Croatia and (1564) Srbija. This pair is of a particular interest to us for the reason that these minor planets are named after our two greatest federal republics, and that (1564) Srbija is the first minor planet discovered at the Belgrade Observatory (in 1936). Moreover, this pair has already been treated by Lazović (1964, 1967), whereby the comparison of the results has been made all the easier. That is why we are going to employ the same orbital elements as those appearing in the two just cited earlier papers. It was with them that we obtained the angle between the orbital planes of these bodies to be $I = 0^\circ\!.2277$. This angle can be found using the formulae given in Lazović and Kuzmanoski (1974). By the graph, given in both of our earlier papers (Lazović, 1964, 1967), we were able to directly determin first the approximate values of the true proximity anomalies of our minor planets, $v_{589,0} = 118^\circ\!.6$, $v_{1564,0} = 105^\circ\!.4$. Thereupon the corresponding approximate values of the eccentric anomalies $E_{589,0} = 116^\circ\!.577$, $E_{1564,0} = 93^\circ\!.277$ were deduced indirectly by means of the above values and (22). With these values we could proceed to the calculus of approximations according to the above derived formulae. Thus we obtained the first corrections to the eccentric anomalies $\Delta E_{589,0} = -0^\circ\!.3062$, $\Delta E_{1564,0} = +0^\circ\!.2120$ and new values $E_{589,1} = 116^\circ\!.2708$, $E_{1564,1} = 93^\circ\!.4890$. By substituting these values of the anomalies in our equations of conditions for the proximity (15) we obtained $f_1 = -0.0000405$, $g_1 = +0.0000519$. The second corrections obtained are $\Delta E_{589,1} = -0^\circ\!.0019$, $\Delta E_{1564,1} = -0^\circ\!.0022$, accordingly the new values of anomalies amount t. $E_{589,2} = 116^\circ\!.2689$, $E_{1564,2} = 93^\circ\!.4868$, while $f_2 = +0.0000059$, $g_2 = -0.0000058$. The third numerical approximation having furnished $\Delta E_{589,2} = 0^\circ\!.0000$, $\Delta E_{1564,2} = 0^\circ\!.0000$, as the final required values of the eccentric anomalies of the proximity positions were adopted those obtained after the second approximation. These values satisfied also the supplementary conditions (8) for the minimum, which were calculated by way of (17) and (18). Next we found, by using (6), the minimum distance between the orbits of the two minor planets (589) Croatia and (1564) Srbija to be $\rho_{\min} = 0.000498$ AU. This result coincided with the one in Lazović (1964, 1967).

The second example is represented by the non-quasicomplanar minor planet pair (1) Ceres and (2) Pallas, whose proximity was found earlier (Lazović, Kuzmanoski, 1980b), but using the true anomalies whereby the comparison of the results was again made easier. The orbital elements were taken from EMP for 1980. We learn from this paper that the angle between the orbital planes of these bodies was $I = 36^\circ\!.65323$. The necessary starting, i. e. approximate values of the eccentric anomalies of the proximity of this pair will be determined in two ways: indirectly and directly. Since the orbital elements in this instance were known with a greater number of decimals, i. e. with higher accuracy, we are going to take advantage and calculate necessary values also with the greater number of decimals, than it was the case in the preceding example where the orbital elements were known with a lower accuracy. We first find indirectly the approximate values of the proximity eccentric anomalies using the approximate values of the proximity true anomalies of these bodies by way of the formulae derived in Lazović (1980). Thus we obtained $v_{1,0} = 213^\circ.32170$, $v_{2,0} = 247^\circ.96828$. Next we calculate, using (22), the starting values $E_{1,0} = 215^\circ.83717$, $E_{2,0} = 260^\circ.99766$. The approximate values of the eccentric anomalies are found directly using the formulae (23), derived by Simovljevitch (1977), keeping in mind that the values of L_i are confined between 0° and 180°. We deem both of the procedures satisfying in this first step of the calculus of proximity, this especially so if the angles between the orbital planes of the celestial bodies under consideration are larger ones. The determination of the approximate values of the eccentric anomalies is preferable in the indirect calculus as a result of the shorter formulae in Lazović (1980), whereas the formulae in Simovljevitch (1977) render it possible the complete calculus to be performed using the same anomalies. On finding the second corrections to the just quoted approximate eccentric anomalies, thus once more after only two approximations by means of (21), the final eccentric anomalies $E_1 = 215^\circ.34175$, $E_2 = 260^\circ.30903$ were derived. Using these values the minimum distance found between the orbits (1) Ceres and (2) Pallas obtained by (6), was $\rho_{min} = 0.0626963$ AU. This value has already been found in Lazović, Kuzmanoski (1980) but in another way, different from that just set out. Accordingly, this procedure proved effective in both instances.

The above derived equations in which the eccentric anomalies are the arguments furnish an additional possibility of the calculus of proximity of the elliptic orbits of celestial bodies, besides that offered by the equations wherein the true anomalies are arguments, Lazović (1967). However, these equations are simpler than those in which true anomalies are figuring as they are shorter. This is the reason of their being more suited both to individual proximity determination, when one is in possession a computer of rather limited capacity, and to serial investigations in view of the efficacity they offer.

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